Christ Church Grammar School

2016
UNIT TEST 5

MATHEMATICS SPECIALIST Year 12

Section One:
Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

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Question 1 (4 marks)

Find the value of c, where $c \in \mathbb{R}$, such that the curve defined by

$$y^2 + \frac{3e^{(x-1)}}{x-2} = c$$

has a gradient of 2 where $x = 1$.

Question 2 (6 marks)

A container of water is heated to boiling point (100 ºC) and then placed in a room that has a constant temperature of 20 ºC. After five minutes the temperature of the water is 80 ºC.

(a) Use Newton's law of cooling

$$\frac{dT}{dt} = -k(T - 20)$$

where T ºC is the temperature of the water at time t minutes after the water is placed in the room, to show that $e^{-5k} = \frac{3}{4}$. (3 marks)

(b) Determine the temperature of the water 10 minutes after it is placed in the room. (3 marks)

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Question 3 **(5 marks)**

The equation of motion for a particle moving in simple harmonic motion is given by

$$\frac{d^2x}{dt^2} = -k^2x$$

where k is a positive constant, x is the displacement of the particle and t is time.

Show that $v^2 = k^2(a^2 - x^2)$, where v is the velocity and a is the amplitude of the motion.

End of questions

Christ Church Grammar School

2016
UNIT TEST 5

MATHEMATICS SPECIALIST Year 12

Section Two:
Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.

2. Answer all questions.

3. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

4. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 4 (5 marks)

The logistic model for growth can be described algebraically by a differential equation of the form

$$\frac{dy}{dt} = ay - by^2, \quad \text{with } a > 0 \text{ and } b > 0.$$

Use calculus to show that

$$y = \frac{a}{b + ce^{-at}}, \quad \text{where } c \text{ is some constant}$$

satisfies the above-mentioned differential equation.

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Question 5 **(3 marks)**

Euler realised that we can use the incremental formula

$$\delta y \approx \frac{dy}{dx}\delta x$$

where δx is the step size in the values of x, to iteratively find approximate values of y.

Euler basically realised that as the x-value changes from x to $x + \delta x$, the y-value will approximately change from y to $y + \delta y$.

Now consider the differential equation

$$\frac{dy}{dx} = \frac{10}{x - 10}$$

with initial values $x = 0$ and $y = 5$.

Use Euler's method with a step size of 0.5 in the values of x to determine an approximate value of y when $x = 1.5$.

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Question 6 **(5 marks)**

A body moves such that its displacement from an origin O at time t seconds is x metres, where $x = 5\cos(3t) - 2\sin(3t)$.

The displacement x can be written as $x = a\sin(3t + b)$, $a > 0$.

(a) Calculate the value of a. (3 marks)

(b) Determine a possible value of b. (2 marks)

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Question 7 (12 marks)

An object on the surface of a liquid is released at time $t = 0$ and immediately sinks.
Let x be the displacement in metres in a downward direction from the surface at time t seconds.

The equation of motion is given by

$$\frac{dv}{dt} = 10 - \frac{v^2}{40}$$

where v is the velocity of the object.

(a) (i) Show that, for $a > t$, $\frac{d}{dt}\left(\ln\left(\frac{a+t}{a-t}\right)\right) = \frac{2a}{a^2 - t^2}$ (3 marks)

(ii) Hence, or otherwise, show that $v = \frac{20(e^t - 1)}{e^t + 1}$ (4 marks)

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(b) Use $\frac{dv}{dt} = v\frac{dv}{dx}$ to show that $x = 20\ln\left(\frac{400}{400 - v^2}\right)$ (3 marks)

(c) How far does the object sink in the first 4 seconds? (2 marks)

See next page

Question 8 (5 marks)

A student carried out a simulation to select a random sample of 100 packets of nails and to record the mean length of the 40 nails in each packet. The nails were assumed to have a mean length of 25 mm and a standard deviation of 2.5 mm.

(a) Describe the expected features of a frequency graph showing the distribution of the 100 sample means. (3 marks)

(b) Sketch a possible frequency graph with the features described in part (a). (2 marks)

End of questions